NEUTRON CHARGE DISTRIBUTION

by

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UNITED STATES
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THESIS

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ABSTRACT

The effects of a neutron charge distribution are considered. It is suggested that the charge distribution is of the form

$$\rho_n(R) = \frac{1}{c^3\pi^{3/2}} e^{-\frac{R^2}{c^2}} - \frac{1}{f^3\pi^{3/2}} e^{-\frac{R^2}{f^2}} .$$

The effects of this charge distribution for various values of the parameters c and f, on the mean square radius of carbon, and on the charge form factor of carbon at high q are shown. The effect of the neutron becomes appreciable in carbon above the diffraction minimum. The neutron in this range of q^2 adds approximately 50% to the carbon form factor.

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1. INTRODUCTION

A review of current literature concerning the nuclear charge distribution shows that the effect of the neutron is largely ignored except in deuteron studies. This paper proposes to suggest what effect, if any, a finite neutron will have on the distribution of charge in any nucleus.

Before the charge distribution in a nucleus can be calculated, the wave functions describing the motion of the nucleons and the nucleons' internal charge distributions must be known. The proton charge and magnetic moment distributions and the neutron magnetic moment distribution are known. The neutron charge distribution is not well known, however.

If wave functions are known, then the neutron charge distribution may be related to the measured nuclear charge distribution. In this paper the diffraction minimum observed in electron scattering by a C^{12} is used to determine the nucleon wave functions.

2. THE NUCLEAR FORM FACTOR

In order to develop a theory for the charge distribution of a nucleus, it is assumed that each nucleon maintains its identity in the nucleus. This assumption leads to models which successfully predict various experimental results. The assumption implies that each nucleon in the nucleus may be described independently of the other nucleons. The nuclear wave functions may be written as follows:

$$(2\text{-}1) \quad \Psi(\vec{r}_1, \cdots, \vec{r}_A) = \frac{1}{A!} \begin{vmatrix} \psi_1'(\vec{r}_1) & \psi_1'(\vec{r}_2) & \cdots\cdots & \psi_1'(\vec{r}_A) \\ \psi_2'(\vec{r}_1) & & & \\ \vdots & & & \\ \psi_A'(\vec{r}_1) & & & \psi_A'(\vec{r}_A) \end{vmatrix}$$

$\vec{r}_i$ describes the position of the center of mass of the i^{th} particle. The ψ' implicitly include a spatial wave function, hereafter referred to as ψ, spin and isospin functions. All of these functions are orthonormal. The use of the othonormality will be implicit in what follows. The total wave function, Ψ , is also normalized and is antisymmetric on interchange of any two nucleons (a consequence of the nucleons being fermions).

The purpose of this paper is to examine the nuclear charge distribution $\rho(R)$. This cannot be measured directly. What can be measured is the differential cross section for scattering electrons, $\frac{d\sigma}{d\Omega}$.

The cross section may be separated into a term due to the charge distribution, and a term due to the magnetic moment of the nucleus [1]. The charge distribution term leads to the charge form factor, $F(q^2)$, which, in Born approximation, is the Fourier transform of the normalized charge distribution.

$$F(q^2) = \frac{1}{Z}\int e^{i\vec{q}\cdot\vec{r}}\rho(\vec{r})\, d^3r \tag{2-2}$$

The Born approximation will be used throughout this paper.

The nuclear charge distribution, $\rho(\vec{R})$, is determined by the nuclear wave functions and the charge distribution of the nucleus.

$$\rho(\vec{R}) = \int \Psi^*(\vec{r}_1,\ldots,\vec{r}_A)\,\hat{\rho}\,\Psi(\vec{r}_1,\ldots,\vec{r}_A)\, d^{3A}r \tag{2-3}$$

where $\hat{\rho}$ is the nucleon charge operator:

$$\hat{\rho} = \sum_{i=1}^{A} f_i(\vec{R}-\vec{r}_i)\,. \tag{2-4}$$

The f_i are the charge operators for the individual nucleons.

Putting equations (2-3) and (2-4) into equation (2-2) yields

$$F(q^2) = \frac{1}{Z}\int e^{i\vec{q}\cdot\vec{R}}\left(\int \Psi^*\sum_{i=1}^{A} f_i(R-r_i)\Psi\, d^{3A}r\right)d^3R \tag{2-5}$$

$$= \frac{1}{Z}\int e^{i\vec{q}\cdot\vec{R}}\left(\sum_{i=1}^{A}\int \Psi^* f_i(\vec{R}-\vec{r}_i)\Psi\, d^{3A}r\right)d^3R\,.$$

Let $\psi_i(r_j)$ be denoted by ψ_{ij}. Note that after integrating over all except the i^{th} coordinate, we have

$$\int \Psi^* f_i(\vec{R}-\vec{r}_i)\Psi\, d^{3A}r = \frac{1}{A}\sum_{j=1}^{A}\int \psi_{ji}^* f_i \psi_{ji}\, d^3r_i\,. \tag{2-6}$$

We see from equation (2-5) and (2-6) that, if the single particle wave functions are orthogonal, the charge distributions for all of the protons are equal, those of all the neutrons are equal and the charge distributions are additive.

Using (2-6) in (2-5) implies

$$\text{(2-7)} \quad F(q^2) = \frac{1}{Z}\int e^{i\vec{q}\cdot\vec{R}} \sum_{i=1}^{A}\left(\frac{1}{A}\sum_{j=1}^{A}\int \psi_{ji}^{*} f_i \psi_{ji}\, d^3r_i\right) d^3R$$

$$= \frac{1}{Z}\sum_{i=1}^{A}\frac{1}{A}\sum_{j=1}^{A}\int e^{i\vec{q}\cdot\vec{R}} \psi_{ji}^{*} f_i(\vec{R}-\vec{r}_i)\psi_{ji}\, d^3r_i d^3R.$$

The relation between $\vec{R}$ and $\vec{r}_i$ is $\vec{R} = \vec{r}_i + \vec{r}_i{}'$.

Insert the following equation into (2-7):

$$\text{(2-8)} \quad \int \delta(\vec{r}_i{}' - (\vec{R}-\vec{r}_i))\, d^3r_i' = 1 .$$

Equation (2-7) becomes

$$\text{(2-9)} \quad F(q^2) = \frac{1}{Z}\sum_{i=1}^{A}\frac{1}{A}\sum_{j=1}^{A}\int e^{i\vec{q}\cdot\vec{R}} \psi_{ji}^{*} f_i \psi_{ji}\, \delta(\vec{r}_i{}' - (\vec{R}-\vec{r}_i))\, d^3r_i' d^3r_i d^3R .$$

Integrate over $\vec{R}$,

$$\text{(2-10)} \quad F(q^2) = \frac{1}{Z}\sum_{i=1}^{A}\frac{1}{A}\sum_{j=1}^{A}\int e^{i\vec{q}\cdot(\vec{r}_i{}'+\vec{r}_i)} |\psi_{ji}|^2 f_i(\vec{r}_i{}')\, d^3r_i d^3r_i'$$

Separate integrals into one over $\vec{r}_i$ and one over $\vec{r}_i{}'$:

$$\text{(2-11)} \quad F(q^2) = \frac{1}{Z}\sum_{i=1}^{A}\left[\frac{1}{A}\sum_{j=1}^{A}\left\{\left(\int e^{i\vec{q}\cdot\vec{r}_i}|\psi_{ji}|^2 d^3r_i\right)\left(\int e^{i\vec{q}\cdot\vec{r}_i{}'} f_i(\vec{r}_i{}')\, d^3r_i'\right)\right\}\right]$$

$$= \frac{1}{Z}\sum_{i=1}^{A}\left[\left(\int e^{i\vec{q}\cdot\vec{r}_i{}'} f_i(\vec{r}_i{}')\, d^3r_i'\right)\frac{1}{A}\sum_{j=1}^{A}\left(\int e^{i\vec{q}\cdot\vec{r}_i}|\psi_j(\vec{r}_i)|^2 d^3r_i\right)\right].$$

Split the sum into one over protons, $f_i = f_p$, and one over neutrons, $f_i = f_n$

$$F(q^2) = \frac{1}{Z}\left[\sum_{i=1}^{Z}\int e^{i\vec{q}\cdot\vec{r}_i{}'} f_p(\vec{r}_i{}')\, d^3r_i{}' \frac{1}{A}\sum_{j=1}^{A}\left(\int e^{i\vec{q}\cdot\vec{r}_i}|\psi_j(\vec{r}_i)|^2 d^3r_i\right)\right.$$
$$\left. + \sum_{i=Z+1}^{A}\int e^{i\vec{q}\cdot\vec{r}_i{}'} f_n(\vec{r}_i{}')\, d^3r_i{}' \frac{1}{A}\sum_{j=1}^{A}\left(\int e^{i\vec{q}\cdot\vec{r}_i}|\psi_j(\vec{r}_i)|^2 d^3r_i\right)\right].$$

Define

$$\text{(2-12)}\quad \begin{cases} F_p = \int e^{i\vec{q}\cdot\vec{r}'} f_p(\vec{r}')\, d^3r' \\ F_n = \int e^{i\vec{q}\cdot\vec{r}'} f_n(\vec{r}')\, d^3r' \end{cases}$$

$$\text{(2-13)}\quad F_j \equiv \int e^{i\vec{q}\cdot\vec{r}} |\psi_j(\vec{r})|^2 d^3r .$$

Since we have integrated over $\vec{r}$, $F_{ij} = F_j$ for all i. Use (2-12) and (2-13) in equation (2-11).

$$\text{(2-14)}\quad F(q^2) = \frac{1}{Z}\left[\sum_{i=1}^{Z} F_p \frac{1}{A}\sum_{j=1}^{A} F_j + \sum_{i=Z+1}^{A} F_n \frac{1}{A}\sum_{j=1}^{A} F_j\right.$$
$$= F_p \frac{1}{A}\sum_{j=1}^{A} F_j + \frac{N}{Z} F_n \frac{1}{A}\sum_{j=1}^{A} F_j .$$

Define

$$\text{(2-15)}\quad F_{cm-A} \equiv \frac{1}{A}\sum_{j=1}^{A} F_j .$$

Then write equation (2-14) as

$$\text{(2-16)}\quad F(q^2) = \left[F_p + \frac{N}{Z} F_n\right] F_{cm-A} .$$

The nuclear form factor has been separated into three parts:

(1) F_p, the form factor of the proton

(2) F_n, the form factor of the neutron, and

(3) F_{cm-A}, a form factor due to the distribution of the centers of mass of the nucleons.

F_{cm-A} is valid for any nucleon distribution, $\rho(\vec{R})$; however, $\rho(\vec{R})$ may be expanded (since any function of angle may be expanded in terms of spherical harmonics) as follows:

$$\rho(\vec{R}) = \sum_{\ell, m=0}^{\infty} a_{\ell m}(R)\, Y_{\ell m}(\theta,\phi) \tag{2-18}$$

$$= \rho_0(R) + \sum_{\ell \geq 1, m=0}^{\infty} a_{\ell m}(R)\, Y_{\ell m}(\theta,\phi)$$

where ρ_0 is $\rho(\vec{R})$ averaged over all angles or the spherically symmetric part of $\rho(\vec{R})$.

$F(q^2)$ may be expanded in terms of the multipole expansion of the charge distribution. This yields an electric monopole term plus higher order multipole terms in the form factor. The simplest nuclei to analyze are then the spherically symmetric nuclei, since these have only the electric monopole term. All further work in this paper will deal only with this type of nucleus.

3. MEAN SQUARE RADIUS DETERMINATION

A quantity which is experimentally easy to determine is the mean square radius. We show below that, theoretically, this radius may be expressed in terms of the particle wave functions and in terms of the charge distributions of these particles.

The form factor for a spherically symmetric charge distribution $\rho(r)$ is

$$F(q^2) = \int e^{i\vec{q}\cdot\vec{r}}\rho(r)\, d^3r \tag{3-1}$$

$$= \frac{4\pi}{q}\int dr\; r\sin(qr)\rho(r) \;.$$

We have on expanding sin (qr):

$$F(q^2) = 4\pi\int \rho(r)\left(1 - \frac{q^2r^2}{3!} + \cdots + \frac{(-1)^N q^{2N} r^{2N}}{(2N+1)!}\right) r^2 dr \tag{3-2}$$

$$= \int \rho(r)\left(1 - \frac{q^2r^2}{3!} + \cdots + \frac{(-1)^N q^{2N} r^{2N}}{(2N+1)!}\right) d^3r$$

$$= 1 - \frac{q^2\langle r^2\rangle}{3!} + \frac{q^4\langle r^4\rangle}{5!} + \cdots + \frac{(-1)^N q^{2N}\langle r^{2N}\rangle}{(2N+1)!} \;.$$

This implies that

$$\left.\frac{\partial F(q^2)}{\partial q^2}\right)_{q^2=0} = -\frac{\langle r^2\rangle}{6} \quad \Rightarrow \tag{3-3}$$

$$\langle r^2\rangle = -6\left.\frac{\partial F(q^2)}{\partial q^2}\right)_{q^2=0} \;.$$

From equation (2-16) we have

$$F(q^2) = \left[F_p + \frac{N}{Z}F_N\right]F_{cm-A} \;. \tag{2-16}$$

Using this in equation (3-3) yields

$$\text{(3-4)} \quad \langle r^2 \rangle = -6 \left.\frac{\partial F(q^2)}{\partial q^2}\right)_{q^2=0} = -6\left[\frac{\partial F_p}{\partial q^2} + \frac{N}{Z}\frac{\partial F_n}{\partial q^2} + \frac{\partial F_{cm-A}}{\partial q^2}\right]_{q^2=0}$$

$$= \langle r_p^2 \rangle + \frac{N}{Z}\langle r_n^2 \rangle + \langle r_{cm-A}^2 \rangle \; .$$

In equation (3-4) we have used

$$\left.F_p\right)_{q^2=0} = 1 \; , \quad \left.F_{cm-A}\right)_{q^2=0} = 1 \quad \text{and} \quad \left.F_n\right)_{q^2=0} = 0 \; .$$

The directly measurable quantities in equation (3-4) are the mean square radii of the nucleus and of the proton. The mean square center of mass radius is dependent on A and the configuration of the nucleons in the nucleus. This configuration may be different for different elements of equal A.

It is thus possible to write the nuclear form factor in terms of the nucleon form factors and nucleon wave functions which describe the states of the nucleons. See equation (2-16). However, starting with experimental data on the nuclear form factor, it is impossible to separate the contributions due to the nucleon charge distributions and that due to the nucleon motion unless a model for the nuclear or nucleon structure is assumed. That is, the analysis will be model dependent. The validity of any model is measured by how well it agrees with experiment.

4. POSSIBLE NUCLEON CHARGE DISTRIBUTIONS

There has been a great deal of work done on the proton which suggests that it has a gaussian charge distribution [2].

$$(4\text{-}1) \quad f_p(r) = \frac{1}{d^3 \pi^{3/2}} e^{-\frac{r^2}{d^2}}$$

The form factor corresponding to this charge distribution is

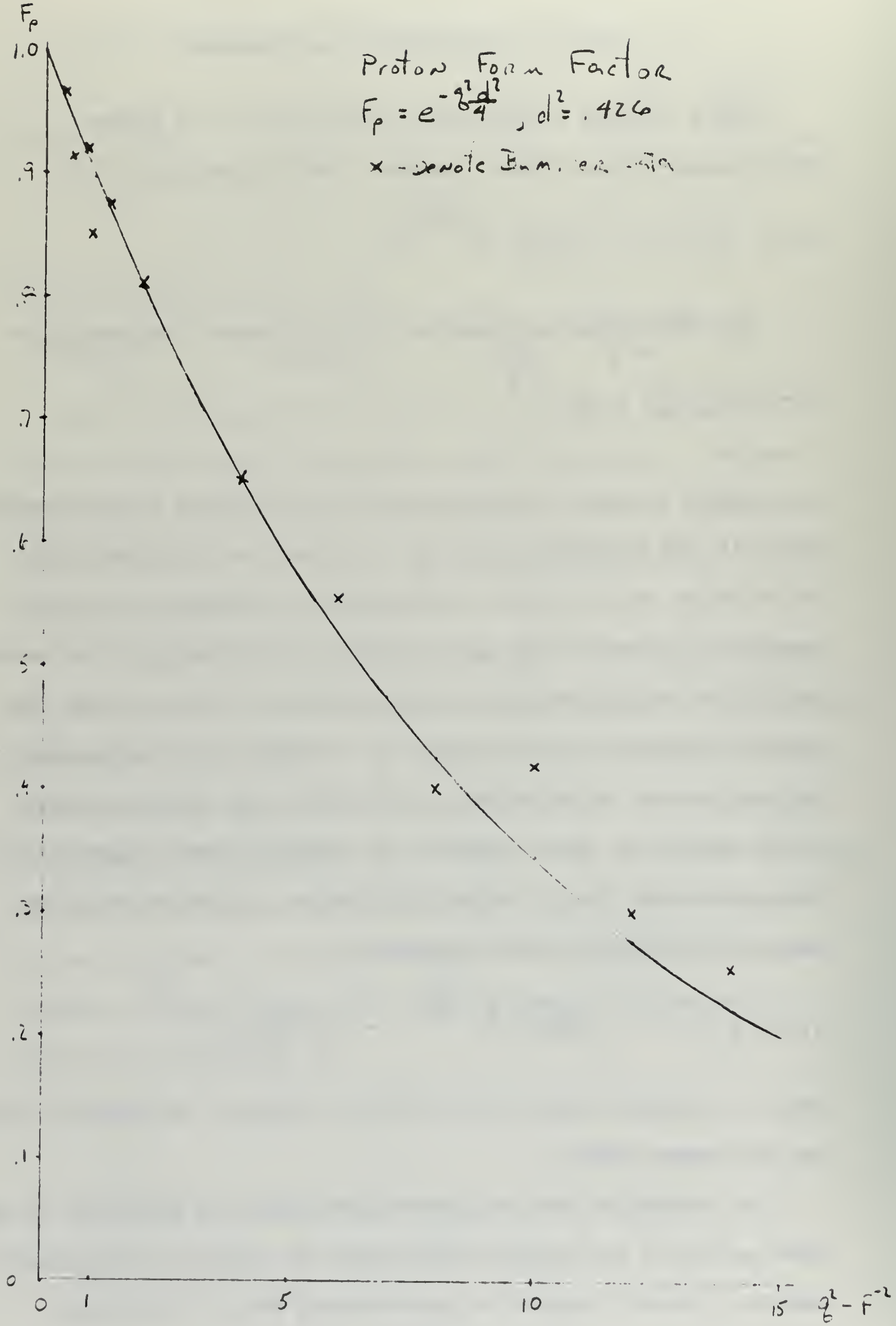
$$(4\text{-}2) \quad F_p(q^2) = e^{-\frac{q^2 d^2}{4}}$$

Figure 1 shows a comparison of this form factor to experimental data [2]. We assume here that f_p is correct, and that the charge distribution and the matter distribution are identical. With this assumption it seems valid that the matter distribution in the neutron could also be distributed in the same manner. That is to say, the matter distribution of the neutron is a gaussian with approximately the same radius as the proton. It is known that the total charge of the neutron is zero. However, its magnetic moment suggests that the neutron does have a charge distribution. By analogy with the proton the following form is appealing:

$$(4\text{-}3) \quad f_n(r) = \frac{1}{c^3 \pi^{3/2}} e^{-\frac{r^2}{c^2}} - \frac{1}{f^3 \pi^{3/2}} e^{-\frac{r^2}{f^2}}$$

This is a double gaussian, each with unit charge. The negative charge has the larger radius.

Our assumption that the matter distribution of the proton is the same as that of the neutron implies that the radius of the neutron's negative (larger) gaussian is approximately that of the proton.



F_p
Proton Form Factor
$F_p = e^{-q^2 d^2/4}$, $d^2 = .426$
x - denote [illegible] data
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
1
5
10
15
$q^2 - F^{-2}$

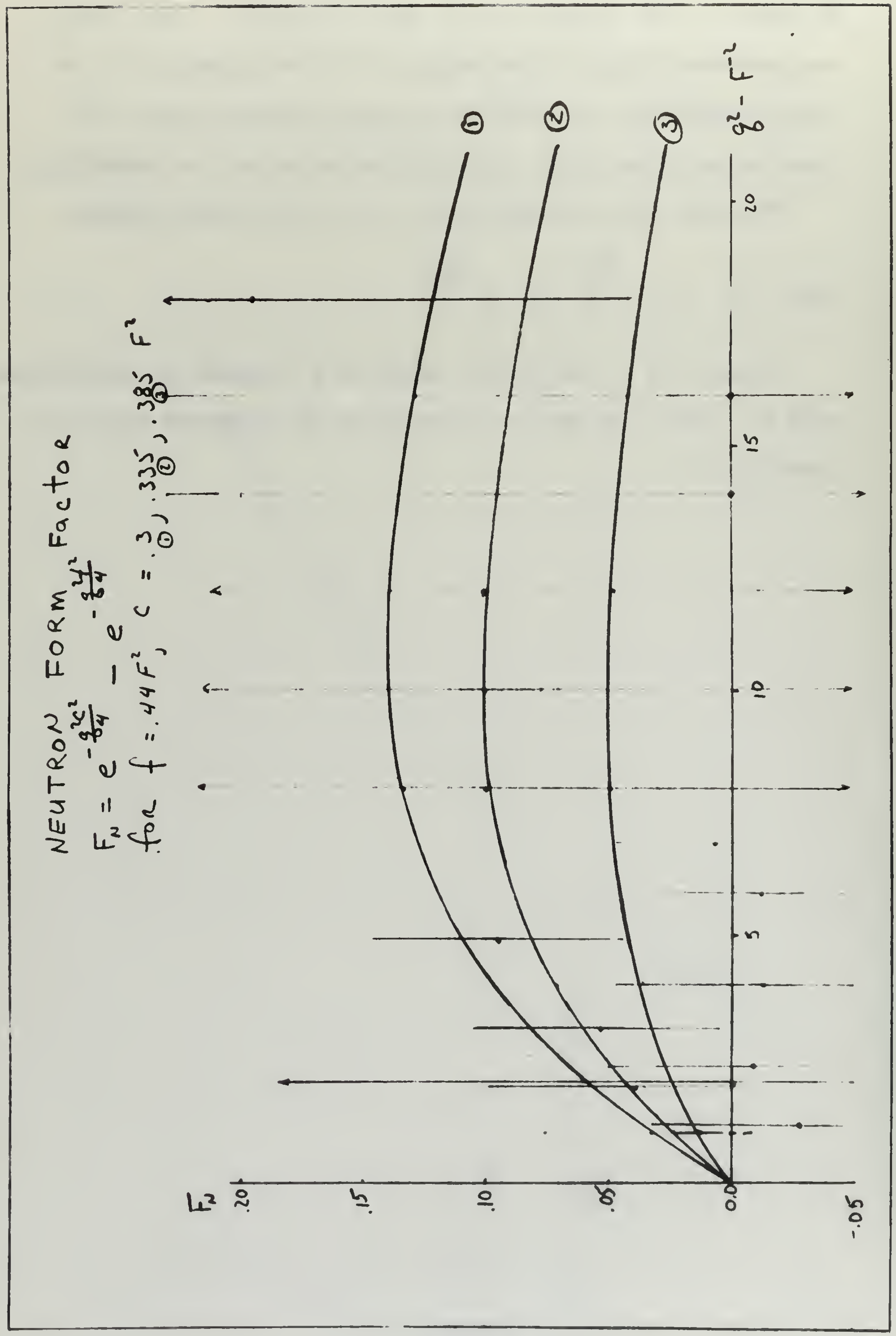
Figure 1

We take f = .44F (slightly larger than the proton). This leaves one parameter to fix in the neutron. The free parameter will be fixed in this paper by using the harmonoc oscillator model and experimental carbon data. (For alternative method see Appendix C.)

The form factor correspond to f_n is, in Born approximation

$$F_n = e^{-\frac{q^2c^2}{4}} - e^{-\frac{q^2f^2}{4}} \quad \text{(4-4)}$$

Figure 2 is F_n for various values of c^2 compared to experimental data [3]. This data does not deny any of the suggested curves, but favors $F_n = 0$.



NEUTRON FORM Factor
$F_N = e^{-\frac{q^2c^2}{64}} - e^{-\frac{q^2f^2}{64}}$
for $f = .44F^2$, $c = .3, .335, .385\ F^2$
F_N
.20
.15
.10
.05
0.0
-.05
1
5
10
15
20
$q^2 - F^{-2}$

Figure 2

5. HARMONIC OSCILLATOR WAVE FUNCTIONS

In order to determine $\langle r_n^2 \rangle$ we are forced to assume a model to predict $\langle r_{cm}^2 \rangle$. The harmonic oscillator potential is a simple potential which appears to agree well with experiment. This theory has the added advantage that its parameters are fixed independently of the mean square radius of the charge distribution.

Consider non-interacting particles in a spherically symmetric harmonic oscillator potential well:

$$V(r) = \frac{1}{2} m\omega^2 r^2 . \tag{5-1}$$

The stationary states or orbitals occupied by the particles are solutions of Schrodinger's equation.

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi(\vec{r}) = E\,\psi(\vec{r}) . \tag{5-2}$$

Using equation (5-1), and defining $a^4 = \frac{y_n^2}{m^2 w^2}$, equation (5-2) can be written as

$$\frac{1}{2} \frac{\hbar^2}{m} \left[-\nabla^2 + \frac{r^2}{a^4} \right] \psi(\vec{r}) = E\,\psi(\vec{r}) \tag{5-3}$$

(5-4) where

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2}{\partial\phi^2} .$$

Assume

$$\psi(\vec{r}) = R(\vec{r})\, Y(\theta,\phi) . \tag{5-5}$$

Using equations (5-4) and (5-5) in equation (5-3)

$$(5\text{-}6)\quad \frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2mr^2}{\hbar^2}\left(E - V(r)\right) =$$

$$-\frac{1}{Y}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] =$$

$$\ell(\ell+1)$$

The solutions to the angular equation are the spherical harmonics. (See Appendix B.)

For convenience in solving the radial equation, define

$$(5\text{-}7)\quad R(r) = \frac{u(r)}{r}$$

The radial portion of equation (5-6) may be written as

$$(5\text{-}8)\quad \frac{d^2u(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E - \frac{1}{2}m\omega^2r^2 - \frac{\ell(\ell+1)\hbar^2}{2mr^2}\right)u(r) = 0 .$$

The solution of equation (5-8) and equation (5-7) yields: (See Appendix B.)

$$(5\text{-}9)\quad R_{N\ell} = r^\ell e^{-\frac{1}{2}\frac{r^2}{a^2}} \sum_{K=0}^{N} C_K\left(\frac{1}{a^2}\right)^K r^{2K} .$$

These equations also yield:

$$(5\text{-}10)\quad E_\Lambda = \hbar\omega\left[2n + \ell + \frac{3}{2}\right]$$

Where Λ = 2N + 1 denotes the energy levels. The harmonic wave functions are then (See Appendix B.)

$$(5\text{-}11)\quad \psi_{N\ell m} = R_{N\ell}Y_{\ell m} .$$

The above solutions are based on the central potential having a fixed origin. In reality, the central potential should be measured from the mean position of the nucleons. It has been shown that this complication introduces a factor $e^{\frac{q^2 a^2}{4A}}$ into the form factor [4].

From equation (2-4) and the above consideration we have (See Appendix B.)

$$F_{n\ell m} = e^{\frac{q^2 a^2}{4A}} \int e^{i\vec{q}\cdot\vec{r}} \left| \psi_{n\ell m} \right|^2 d^3 r \tag{5-12}$$

From equation (2-16) we have

$$F_{cm-A} = \frac{1}{A} \sum_{i}^{A} F_{(n\ell m)_i} \tag{5-13}$$

If the harmonic oscillator model is applicable and if the nucleon form factors F_n and F_p and the nuclear configuration (i.e., n, l, m must be known for each nucleon) are known, then the total nuclear form factor may be written. For example, for p-shell nuclei ($4 \leq A \leq 16$, $2 \leq Z \leq 8$) [5]:

$$F_{cm-A}(q^2) = \frac{1}{A} \sum_{i}^{A} F_i = \frac{1}{A}\left[4 F_{000} + (A-4) F_{\ldots} \right] \tag{5-14}$$

$$= \frac{1}{A}\left(A - (A-4)\frac{q^2 a^2}{6} \right) e^{-\frac{q^2 a^2}{4}\left(1-\frac{1}{A}\right)}$$

$$= \left(1 - \frac{A-4}{6A} q^2 a^2 \right) e^{-\frac{q^2 a^2}{4}\left(1-\frac{1}{A}\right)}$$

The F_{nlm} from Appendix B have been used. In Carbon $A = 12$, $Z = N = \frac{A}{2}$:

$$F_{cm-A} = \left(1 - \frac{q^2 a^2}{9} \right) e^{-\frac{q^2 a^2}{4}\left(1-\frac{1}{12}\right)}$$

Or the total form factor is

$$F_c(q^2) = (F_p + F_n)\left(1 - \frac{q^2 a^2}{9}\right) e^{-\frac{q^2 a^2}{4}\left(1 - \frac{1}{12}\right)} .$$

F_c has a zero at $q^2 = \frac{q}{a^2}$. Therefore, our cross section measurements should also show a zero at this point. Experimental data show a marked minimum in the cross section which can be used to fix a^2.

6. COMPARISON TO EXPERIMENTAL DATA

In the harmonic oscillator model, the center of mass form factor, F_{cm-A}, is completely determined by the position of the minimum in a plot of $F^2(q^2)$ vs. q^2.

Carbon offers a very good method of fixing the neutron parameters since it is spherically symmetric and the experimental data show a prominent diffraction minimum [6].

$$F_c(q^2) = (F_p + F_n)\left(1 - \frac{q^2 a^2}{9}\right) e^{-\frac{q^2 a^2}{4}\left(1 - \frac{1}{12}\right)} \tag{6-1}$$

$$a^2 = \frac{9}{q^2_{\text{Diffraction Minimum}}} = \frac{9}{3.27\ F^{-2}} = 2.75\ F^2 \tag{6-2}$$

Figure 3 is a comparison of equation (6-1) to experimental values for the neutron parameters of Figure 2 [6]. This figure indicates that the best neutron form factor is given by

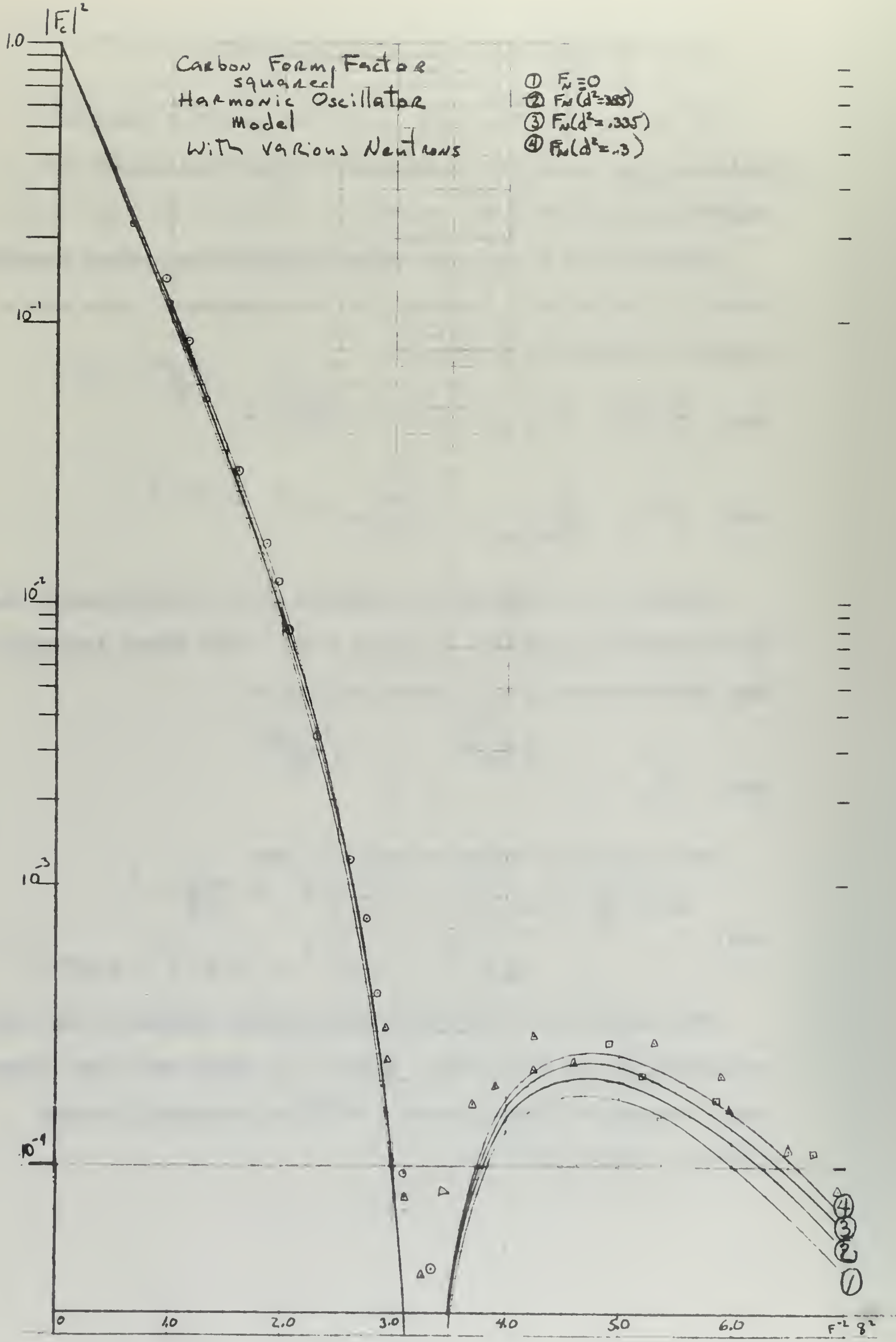
$$F_n = e^{-q^2\frac{(.3)^2}{4}} - e^{-q^2\frac{(.44)^2}{4}} \tag{6-3}$$

.

The mean square radius of carbon is then

$$\begin{aligned} \langle r^2 \rangle_c &= \langle r_p^2 \rangle + \langle r_n^2 \rangle + \frac{49}{24} a^2 \\ &= .64\ F^2 - .21\ F^2 + 5.61\ F^2 = 6.04\ F^2 \end{aligned} \tag{6-4}$$

This radius is, of course, model-dependent because of the manner in which $\langle r^2_{cm-A} \rangle$ has been found. However, it agrees well with current model independent determinations of $\langle r^2 \rangle c$ at Darmstadt in which $\langle r^2 \rangle c = .587 \pm .19\ F^2$ [7].



$|F_c|^2$
1.0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Carbon Form Factor squared
Harmonic Oscillator Model
With various Neutrons
① $F_N = 0$
② $F_N (d^2 = .385)$
③ $F_N (d^2 = .335)$
④ $F_N (d^2 = .3)$
0
1.0
2.0
3.0
4.0
5.0
6.0
F^{-2} q^2

Figure 3

7. DISCUSSION

The preceeding results suggest two alternative approaches for determining the validity of the neutron radius in equation (6-4). First, assume that the nucleons are described by harmonic oscillator wave functions, in which the range parameter a is adjusted to give the experimentally observed diffraction minimum in $F(q^2)$. This allows us to write

$$\frac{N}{Z}\langle r_n^2\rangle = \langle r^2\rangle_A - \langle r_p^2\rangle - \langle r_{cm-A}^2\rangle . \tag{7-1}$$

We would then use the model independent $\langle r^2\rangle_A$ determined at low q^2, the accepted value of $\langle r_p^2\rangle$ determined from H^1 data, and the shell model prediction of $\langle r_{cm-A}^2\rangle$. Extreme accuracy in the measured quantities is required due to the small size of $\langle r_n^2\rangle$.

The second alternative would be to assume that the nucleon center of mass distribution is the same for elements with equal A but different Z. This assumption is based on the charge independence of nuclear forces. The assumption could be checked in a model-dependent way by seeing whether or not their diffraction minima occurred at the same value of q. Measuring the model-independent radii of two elements with the same atomic weight at low q and subtracting the following:

$$
\begin{aligned}
&\langle r^2\rangle_Z = \langle r_p^2\rangle + \frac{N}{Z}\langle r_n^2\rangle + \langle r_{cm-A}^2\rangle \\
-&\left(\langle r^2\rangle_{Z'} = \langle r_p^2\rangle + \frac{N'}{Z'}\langle r_n^2\rangle + \langle r_{cm-A}^2\rangle\right) \\
\hline
&\langle r^2\rangle_Z - \langle r^2\rangle_{Z'} = \left(\frac{N}{Z} - \frac{N'}{Z'}\right)\langle r_n^2\rangle \implies
\end{aligned}
$$

$$\langle r_n^2\rangle = \frac{ZZ'}{A(Z-Z')}\left[\langle r^2\rangle_Z - \langle r^2\rangle_{Z'}\right] . \tag{7-3}$$

Thus $\langle r_n^2 \rangle$ is obtained in terms of only the model independent mean square radii. The assumption, however, that $\langle r^2_{cm-A} \rangle = \langle r^2_{cm-A} \rangle$ would only be valid if the nucleon configurations of the elements were identical. One possibility would be to use chromium-50 and titanium-50. These two nuclei have total angular momentum (J) equal to zero. Therefore, they have only a monopole form factor. Any other pair of mirror nuclei would do equally well.

Both of these methods could be checked by comparing the theoretical form factors to the experimental form factors at high q^2. This is the manner that the neutron parameters were fixed in this paper.

The limitation on the parameter f in the neutron charge distribution (that the neutron matter distribution equals the negative charge distribution) is of questionable validity. If this restriction is removed, and we use instead the electron-neutron interaction condition,

(7-5) $$\left.\frac{\partial F_n}{\partial q^2}\right)_{q^2=0} = .021 \ F^2 \qquad [8], [9],$$

which implies

(7-6) $$\langle r_n^2 \rangle = -.126 \ F^2$$

Then the difference between the two neutron parameters is .084 F^2.

(7-7) $$\left.\frac{\partial F_n}{\partial q^2}\right)_{q^2=0} = \frac{1}{4}\left(c^2 - f^2\right) = -.021 \ F^2 .$$

When this restriction is used and the form factor

$$F_n = e^{-\frac{q^2 c^2}{4}} - e^{-\frac{q^2 f^2}{4}}$$

is again fitted to various experimental data, a good fit is obtained for $c^2 = .09\ F^2$, $f^2 = .174\ F^2$.

This choice of parameters leads to a form factor which gives a smaller radius at low q^2, but at q^2 of about 25 F^{-2} the radius is much greater than with the previous choice of parameters. The form factor of the neutron, with this choice of parameters, becomes about three times that of the proton at $q^2 = 25\ F^{-2}$. This doesn't seem reasonable. The previous selection of neutron parameters is, consequently, preferable.

8. CONCLUSION

Current experimental data point to the possibility that the charge distribution of the neutron has a greater mean square radius than previously thought. It appears that a value of $\langle r_n^2 \rangle = -.2F^2$ is not unlikely. In fact, such a value is supported by current measurements of nuclear radii.

It should be noted that it is presently impossible to separate the effect of the neutron from the effect of the wave functions in currently available experimental data on electron scattering. A modification of the wave functions can give form factors which fit the data over a large range of q without considering any neutron effect.

The approach suggested in this paper has the advantage of simplicity. Simple wave functions and a simple neutron give agreement with $F(q^2)$ data.

If the charge distribution of the neutron can be determined independently (for example, through neutron-electron interactions), then this information could be used in fitting wave functions to experimental data.

APPENDIX A. COMMON INTEGRALS

(A-1) $$\int_{\text{All Space}} e^{iqr\cos\theta} f(r) \sin\theta \, d\theta \, r^2 dr \, d\phi =$$

$$\frac{4\pi}{q} \int_0^\infty r \, dr \, \sin(qr) f(r)$$

(A-2) $$\int_{\text{All Space}} e^{iqr\cos\theta} e^{-\frac{r^2}{a^2}} d^3r = a^3 \pi^{3/2} e^{-\frac{q^2a^2}{4}}$$

(A-3) $$\int_{\text{All Space}} e^{iqr\cos\theta} \left(r^2 e^{-\frac{r^2}{a^2}}\right) d^3r = \frac{3}{2} a^5 \pi^{3/2} \left(1 - \frac{q^2a^2}{6}\right) e^{-\frac{q^2a^2}{4}}$$

(A-4) $$\int_{\text{All Space}} e^{iqr\cos\theta} \left(r^4 e^{-\frac{r^2}{a^2}}\right) d^3r =$$

$$a^7 \pi^{3/2} \left(\frac{15}{4} - \frac{5}{4} q^2 a^2 + \frac{(q^2a^2)^2}{16} \right) e^{-\frac{q^2a^2}{4}}$$

APPENDIX B

HARMONIC OSCILLATOR WAVE FUNCTIONS AND FORM FACTORS

1. Spherical Harmonics

$$Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_{1,\pm 1} = \sqrt{\frac{3}{8\pi}} \sin\theta \, e^{\pm i\phi}$$

$$Y_{20} = \frac{1}{4}\sqrt{\frac{5}{\pi}} \left(3\cos^2\theta - 1\right)$$

$$Y_{2,\pm 1} = \frac{1}{2}\sqrt{\frac{15}{2\pi}} \sin\theta\cos\theta \, e^{\pm i\phi}$$

$$Y_{2,\pm 2} = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \sin^2\theta \, e^{\pm 2i\phi}$$

2. Radial Wave Functions $R_{n\ell} = r^{\ell} e^{-\frac{1}{2}\frac{r^2}{a^2}} \sum_{k=0}^{n} C_k \left(\frac{1}{a^2}\right)^k r^{2k}$

$$\Lambda = 0 \qquad R_{00} = \left[\frac{4}{a^3\sqrt{\pi}}\right]^{\frac{1}{2}} e^{-\frac{1}{2}\frac{r^2}{a^2}}$$

$$\Lambda = 1 \qquad R_{01} = \left[\frac{8}{a^5\, 3\sqrt{\pi}}\right]^{\frac{1}{2}} r\, e^{-\frac{r^2}{2a^2}}$$

$$\Lambda = 2 \begin{cases} R_{10} = \left[\dfrac{6}{a^3\sqrt{\pi}}\right]^{\frac{1}{2}} \left(1 - \dfrac{2}{3}\dfrac{r^2}{a^2}\right) e^{-\frac{r^2}{2a^2}} \\ R_{02} = \left[\dfrac{16}{15\, a^7\sqrt{\pi}}\right]^{\frac{1}{2}} r^2 e^{-\frac{r^2}{2a^2}} \end{cases}$$

3. Harmonic Oscillator Wave Functions $\psi_{n\ell m} = R_{n\ell} Y_{\ell m}$

$$\psi_{000} = \left[\frac{1}{a^3 \pi^{3/2}} \right]^{\frac{1}{2}} e^{-\frac{1}{2}\frac{r^2}{a^2}}$$

$$\psi_{010} = \left[\frac{2}{a^5 \pi^{3/2}} \right]^{\frac{1}{2}} r \cos\theta \; e^{-\frac{r^2}{2a^2}}$$

$$\psi_{01\pm1} = \left[\frac{1}{a^5 \pi^{3/2}} \right]^{\frac{1}{2}} r \sin\theta \; e^{\pm i\phi} \; e^{-\frac{r^2}{2a^2}}$$

$$\psi_{100} = \left[\frac{3}{2 a^3 \pi^{3/2}} \right]^{\frac{1}{2}} \left(1 - \frac{2}{3} \frac{r^2}{a^2} \right) e^{-\frac{r^2}{2a^2}}$$

$$\psi_{020} = \left[\frac{1}{3 a^7 \pi^{3/2}} \right]^{\frac{1}{2}} r^2 e^{-\frac{r^2}{2a^2}} (3\cos^2 - 1)$$

$$\psi_{02\pm1} = \left[\frac{2}{a^7 \pi^{3/2}} \right]^{\frac{1}{2}} r^2 \; e^{-\frac{r^2}{2a^2}} \cos\theta \sin\theta \; e^{\pm i\phi}$$

$$\psi_{02\pm2} = \left[\frac{1}{2 a^7 \pi^{3/2}} \right]^{\frac{1}{2}} r^2 \; e^{-\frac{r^2}{2a^2}} \sin^2\theta \; e^{\pm 2i\phi}$$

4. Electric Monopole Form Factors for Independent Particles

$$F_{000} = e^{-\frac{\xi^2 a^2}{4}\left(1-\frac{1}{A}\right)}$$

$$F_{01m} = \left(1 - \frac{\xi^2 a^2}{6} \right) e^{-\frac{\xi^2 a^2}{4}\left(1-\frac{1}{A}\right)}$$

$$F_{100} = \left(1 - \frac{1}{3}\xi^2 a^2 + \frac{(\xi^2 a^2)^2}{24} \right) e^{-\frac{\xi^2 a^2}{4}\left(1-\frac{1}{A}\right)}$$

$$F_{02m} = \left(1 - \frac{\xi^2 a^2}{3} + \frac{(\xi^2 a^2)^2}{60} \right) e^{-\frac{\xi^2 a^2}{4}\left(1-\frac{1}{A}\right)}$$

APPENDIX C

CLASSICAL MAGNETIC MOMENT DETERMINATION

Classically the magnetic moment m is defined as

(C-1) $$\vec{m} = \frac{1}{2c}\int \vec{r} \times \vec{J}(\vec{r})\, d^3 r$$ [10]

where $\vec{r}$ is the distance from some point of interest and $\vec{J}$ is the current density. For an arbitrary charge distribution, $\rho(\vec{r})$, rotating about an axis with velocity $\omega \hat{k}$

(C-2) $$\vec{J} = (\vec{\omega} \times \vec{r})\,\rho(\vec{r}).$$

Using this in equation (C-1) yields

(C-3) $$\vec{m} = \frac{1}{2c}\int \vec{r} \times \vec{\omega} \times \vec{r}\,\rho(\vec{r})\, d^3 r.$$

Using the vector identity $\vec{r} \times \vec{\omega} \times \vec{r} = (\vec{r} \cdot \vec{r})\vec{\omega} - (\vec{r} \cdot \vec{\omega})\vec{r}$ this becomes

(C-4) $$\vec{m} = \frac{1}{2c}\int (r^2 \vec{\omega} - r^2 \omega \hat{r})\,\rho(\vec{r})\, d^3 r$$

We are interested in finding the magnetic moment for neutrons and protons projected onto the R axis. For a spherically symmetric charge distribution we have

(C-5) $$\vec{m} \cdot \hat{k} = \frac{1}{2c}\left[\int r^2 \omega \rho(r)\, d^3 r - \int r^2 \omega \cos\theta\, \rho(r)\, d^3 r\right]$$

The second term integrates to zero when the angular integration is performed.

(C-6) $$\vec{m} \cdot \hat{k} = \frac{\omega}{2c}\int r^2 \rho(r)\, d^3 r = \frac{\omega}{2c}\langle r^2 \rangle$$

So the magnetic moment about the Z axis is for the proton

$$M_{zp} = \frac{\omega_p}{2c} < r_p^2 > \tag{C-7}$$

and for the neutron

$$M_{zn} = \frac{\omega_n}{2c} < r_n^2 > \tag{C-8}$$

The ratio of the proton moment to the neutron moment is then

$$R = \frac{M_{zp}}{M_{zn}} = -\frac{2.79}{1.91} = \frac{< r_p^2 >}{< r_n^2 >} \tag{C-9}$$

assuming $\omega_n = \omega_p$

This agrees in sign with the value predicted for $< r_n^2 >$ as compared to $< r_p^2 >$ but is only the correct order of magnitude and, therefore, gives neutron parameters larger than those in this paper.

$$< r_n^2 > = \frac{< r_p^2 >}{R} = -\frac{.64}{1.46} = -.439\ F^2. \tag{C-10}$$

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13. ABSTRACT

The effects of a neutron charge distribution are considered. It is suggested that the charge distribution is of the form

$$\rho_n(R) = \frac{1}{c^3 \pi^{3/2}} e^{-\frac{R^2}{c^2}} - \frac{1}{f^3 \pi^{3/2}} e^{-\frac{R^2}{f^2}} .$$

The effects of this charge distribution for various values of the parameters c and f, on the mean square radius of carbon, and on the charge form factor of carbon at high q are shown. The effect of the neutron becomes appreciable in carbon above the diffraction minimum. The neutron in this range of q^2 adds approximately 50% to the carbon form factor.

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